Force Behaviour in Radiation-Dominated Friedmann Universe

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Abstract

We consider radiation-dominated Friedmann universe and evaluate its force four-vector and momentum. We analyse and compare the results with the already evaluated for the matter-dominated Friedmann model. It turns out that the results are physically acceptable.

1   Introduction

It is of cosmological interest to consider a fluid composed of incoherent radiations. The primordial radiation played a dominant role in the early universe for the times smaller than the times of re-combination. We have obtained new insights by considering the force four-vector [1] and momentum [2] for the matter-dominated Friedmann universe. Further insights can be expected by analysing the same quantities for the radiation-dominated Friedmann universe. The procedure adopted for this purpose is the extended pseudo-Newtonian $(e\psi N)$-formalism [1].

Einstein's Theory of Relativity replaces the use of forces in dynamics by what Wheeler calls "geometrodynamics". Paths are bent, not by forces, but by the "curvature of spacetime". In the process the guidance of the intuition based on the earlier dynamics is lost. However, our intuition continues to reside in the force concept, particularly when we have to include other forces in the discussion. For this reason one may want to reverse the procedure of

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General Relativity (GR) and look at the non-linear "force of gravity" which would predict the same bending of the path as predicted by geometry. In the ψN-approach [3] the curvature of the spacetime is "straightened out" to yield a relativistic force which bends the path, so as to again supply the guidance of the earlier, force-based, intuition.

The idea of re-introducing the Newtonian gravitational force into the theory of GR arose in an attempt to deal with the following problem: Gravitation, being non-linear, should dominate over the Coulomb interaction at some, sufficiently small, scale. At what scale would it occur? Whereas this question is perfectly valid in pre-relativistic terms it becomes meaningless in GR. The reason is that gravitation is expressed in purely geometric terms while electromagnetism is not. Thus, in Relativity, gravitation possesses a very different status than the other forces of Nature. Our physical intuition for the other interactions, nevertheless, rests on the concept of forces. To deal with gravity and other forces together, we must either express the other forces geometrically, as in the Kaluza-Klein theories, or express gravitation in the same terms as the other forces. We will follow the latter alternative as the simpler program to implement.

The procedure adopted converts an idealised operational definition of the gravitational force (via the tidal force) into a mathematical formulation. The relativistic analogue of the Newtonian gravitational force which gives the relativistic expression for the tidal force in terms of the curvature tensor is called the $e\psi N$-force. The quantity whose proper time derivative gives the $e\psi N$-force is the four-vector momentum of a test particle. The spatial components of this vector give the momentum imparted to a test particle as defined in the preferred frame $(g_{0i} = 0 = g_{ab,0i})(a, b = 0, 1, 2, 3)$. The plan of the paper is as follows: In the next section we shall give a brief description of the formalism employed. In section three, we shall apply the formalism to evaluate force four-vector and momentum four-vector for radiation-dominated Friedmann models. In the last section we shall conclude the results achieved.

2   The $e\psi N$-Formalism

The basis of the formalism is the observation that the tidal force, which is operationally determinable, can be related to the curvature tensor by

$$F_T^\mu = mR_{\nu\rho\pi}^\mu t^\nu l^\rho t^\pi, \quad (\mu, \nu, \rho, \pi = 0, 1, 2, 3), \tag{1}$$

where m is the mass of a test particle, $t^\mu = f(x)\delta_0^\mu$, $f(x) = (g_{00})^{-1/2}$ and l^μ is the separation vector. l^μ can be determined by the requirement that the tidal force have maximum magnitude in the direction of the separation vector. Choosing a gauge in which $g_{0i} = 0$ (similar to the synchronous coordinate system [4]) in a coordinate basis. We further use Riemann normal coordinates (RNCs) for the spatial direction, but not for the temporal direction. The reason for this difference is that both ends of the accelerometer are spatially free, i.e., both move and do not stay attached to any spatial point. However, there is a "memory" of the initial time built into the accelerometer in that the zero position is fixed then. Any change is registered that way. Thus "time" behaves very differently from "space".

The relativistic analogue of the Newtonian gravitational force called the ψN gravitational force, is defined as the quantity whose directional derivative along the accelerometer, placed along the principal direction, gives the extremised tidal force and which is zero in the Minkowski space. Thus the $e\psi N$ force, F_μ, satisfies the equation

$$F_T^{*\mu} = l^\nu F_{;\nu}^\mu, \tag{2}$$

where $F_T^{*\mu}$ is the extremal tidal force corresponding to the maximum magnitude reading on the dial. Notice that $F_T^{*0} = 0$ does not imply that $F^0 = 0$. With the appropriate gauge choice and using RNCs spatially, Eq.(2) can be written in a space and time break up as

$$l^i(F_{,i}^0 + \Gamma_{ij}^0 F^j) = 0, \tag{3}$$

$$l^j(F_{,j}^i + \Gamma_{0j}^i F^0) = F_T^{*i}. \tag{4}$$

A simultaneous solution of the above equations can be found by taking the ansatz

$$F_0 = -m\left[\{\ln(Af)\}_{,0} + g^{ik}g_{jk,0}g^{jl}g_{il,0}/4A\right], \quad F_i = m(\ln f)_{,i}, \tag{5}$$

where $A = (\ln\sqrt{-g})_{,0}$, $g = det(g_{ij})$. This force formula depends on the choice of frame, which is not uniquely fixed.

The new feature of the $e\psi N$ force is its zero component. In special relativistic terms, which are relevant for discussing forces in a Minkowski space, the zero component of the four-vector force corresponds to a proper rate of change of energy of the test particle. Further, we know that in general

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an accelerated particle either radiates or absorbs energy according as $\frac{dE}{dt}$ is less or greater than zero. Thus F_0, here, should also correspond to energy absorption or emission by the background spacetime. Infact we could have separately anticipated that there should be energy non-conservation as there is no timelike isometry. In that sense F_0 gives a measure of the extent to which the spacetime lacks isometry.

Another way of interpreting F_0 is that it gives measure of the change of the "gravitational potential energy" in the spacetime. In classical terms, neglecting this component of the $e\psi N$ force would lead to erroneous conclusions regarding the "energy content" of the gravitational field. Contrariwise, including it enables us to revert to classical concepts while dealing with a general relativistically valid treatment. It can be hoped that this way of looking at energy in relativity might provide a pointer to the solution of the problem of definition of mass and energy in GR.

The spatial component of the $e\psi N$ force F_i is the generalisation of the force which gives the usual Newtonian force for the Schwarzschild metric and a "$\frac{Q^2}{r^3}$" correction to it in the Riessner-Nordstrom metric. The ψN force may be regarded as the "Newtonian fiction" which "explains" the same motion (geodesic) as the "Einsteinian reality" of the curved spacetime does. We can, thus, translate back to Newtonian terms and concepts where our intuition may be able to lead us to ask, and answer, questions that may not have occurred to us in relativistic terms. Notice that F_i does not mean deviation from geodesic motion.

The quantity whose proper time derivative is F_μ gives the momentum four-vector for the test particle. Thus the momentum four-vector, p_μ, is [5]

$$p_\mu = \int F_\mu dt. \tag{6}$$

The spatial components of this vector give the momentum imparted to test particles as defined in the preferred frame (in which $g_{0i} = 0$).

3   Radiation-Dominated Friedmann Universe

The metric describing the standard model [6,7] is the Friedmann metric

$$ds^2 = dt^2 - [\frac{dr^2}{1 + kr^2/a^2(t)} + r^2 d\Omega^2], \tag{7}$$

where k can take the values $-1, 0$ or 1. In these three cases a new variable, χ, can be used in place of r

$$open(k = -1), \quad r = a(t)\sinh\chi \tag{8}$$

$$flat(k = 0), \quad r = a(t)\chi \tag{9}$$

$$closed(k = +1), \quad r = a(t)\sin\chi \tag{10}$$

to give the metric

$$ds^2 = dt^2 - a^2(t)[d\chi^2 + \sigma^2(\chi)d\Omega^2], \tag{11}$$

where χ is the hyperspherical angle, $\sigma(\chi) = \sinh\chi, \chi, \sin\chi$ as for $k = -1, 0, 1$ and $a(t)$ is a scale parameter. For radiation-dominated universe, it is given [8] by

$$a(t) = a_1 \sinh\eta, \quad t = a_1(\cosh\eta - 1)(0 \le \eta < \infty), \quad k = -1 \tag{12}$$

$$a(t) = \sqrt{2a_1 t}, \quad k = 0 \tag{13}$$

$$a(t) = a_1 \sin\eta, \quad t = a_1(1 - \cos\eta)(0 \le \eta \le 2\pi), \quad k = +1 \tag{14}$$

or by [9]

$$a(t) = \sqrt{2a_1 t + t^2} \quad (k = -1), \quad \sqrt{2a_1 t} \quad (k = 0), \quad \sqrt{2a_1 t - t^2} \quad (k = +1) \tag{15}$$

The $e\psi N$-force, for the Friedmann models, is given [1] as

$$F_0 = -m\frac{\ddot{a}}{\dot{a}}, \quad F_i = 0, \tag{16}$$

where dot denotes differentiation with respect to the coordinate time t. The corresponding p_0 and momentum p_i, imparted to test particle, are given [2] by

$$p_0 = -m\ln(c\dot{a}), \quad p_i = constant, \tag{17}$$

where c is an integration constant.

Now we evaluate the $e\psi N$-force and momentum for all the three cases of radiation-dominated Friedmann models.

For the flat model, it will be

$$F_0 = \frac{m}{2t}, \quad F_i = 0 \tag{18}$$

Thus F_0 is proportional to t^{-1} and hence F_0 goes to infinity as t approaches to zero and it tends to zero when t tends to infinity. Since F_0 is positive, it corresponds to the energy absorption [1] by the background spacetime. The corresponding p_0 and p_i will become

$$p_0 = m \ln(\frac{\sqrt{2t}}{c\sqrt{a_1}}), \quad p_i = constant \tag{19}$$

For the open Friedmann model, Eq.(5) gives

$$F_0 = -\frac{m}{a_1 sinh^2 \eta cosh \eta}, \quad F_i = 0 \tag{20}$$

Hence F_0 goes as t^{-3} for large values of t. Consequently, the quantities p_0 and p_i turn out to be

$$p_0 = m \ln(\frac{\tanh \eta}{c}), \quad p_i = constant. \tag{21}$$

For the closed Friedmann universe, it will be

$$F_0 = \frac{m}{a_1 sin^2 \eta cos \eta}, \quad F_i = 0 \tag{22}$$

Thus F_0 turns out to be infinite for $\eta = 0, \pi, 2\pi$. The corresponding p_0 and p_i

$$p_0 = m \ln(\frac{tan \eta}{c}), \quad p_i = constant. \tag{23}$$

4 Conclusion

It has been shown that the $e\psi N$-formalism, which had been useful in providing insights for matter-dominated Friedmann universe, can be used for radiation-dominated Friedmann universe. Further, we have evaluated the momentum 4-vector. It is shown that the behaviour of flat Friedmann universe is same for both matter and radiation-dominated universes. The temporal component of force goes to zero for large values of t. However, the spatial components are zero in all the three cases as there is no gravitational

source present in the Friedmann universe. For the open and closed radiation-dominated universes, the behaviour of the temporal component of the force is similar and it goes to infinity as η tends to 0. It is worth mentioning here that the force does not remain finite even for closed model at $\eta = \pi$ though it was finite for the matter-dominated universe [1]. This is the difference between the matter and radiation-dominated models. The reason is that radiation-dominated era is the early universe and at that time everything is same and there is no concept of closedness. Thus one justifies the infinite rate of change of energy at the early universe.

The momentum 4-vector shows that the temporal component, which gives energy of a test particle, is infinite in all the three cases. However, the momentum of a test particle turns out to be constant in each case and this exactly coincides with the matter-dominated model already evaluated [2].

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